CHANNEL EQUALIZATION FOR SYNCHRONIZATION OF IKEDA MAPS

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ABSTRACT

Many communication systems based on the synchronism of chaotic systems have been proposed in the literature. However, due to the lack of robustness of chaos synchronization, even minor channel imperfections are enough to hinder communication. In this paper, we propose an adaptive equalization scheme to recover a binary sequence modulated by a chaotic signal, which in turn is generated by Ikeda maps. The proposed scheme employs the normalized least-mean-squares (NLMS) algorithm with a modification to enable chaotic synchronization even when the communication channel is not ideal. Simulation results show that the modified NLMS can successfully equalize the channel in different scenarios.

***Index Terms*—** Chaotic synchronization, chaotic communication system, adaptive equalizers, LMS algorithm.

1. INTRODUCTION

In the last two decades, the feasibility of communication systems based on the synchronism of chaotic systems has been theoretically and experimentally investigated (e.g. [1–4]). A chaotic system deterministically generates trajectories in the state space that are aperiodic, limited and present dependence on initial conditions [5]. Therefore, chaotic signals have been proposed as broadband information carriers with the potential of providing high level of privacy in data transmission [6, 7].

Recently, some works with a practical approach using chaos in the signal level have appeared, mainly in the optical communication domain (see, e.g., [6]). This is somewhat natural since chaotic generators can be easily created using the intrinsic nonlinear properties of lasers. This fact was exploited in [6], where a high-speed long-distance communication based on chaos synchronization was demonstrated over a commercial fibre-optic channel. The continuous-time system of [6] considers a chaotic modulation in which the message is fed back into the chaotic signal generator (CSG). In this context, it is common to model the CSG using variants of the Ikeda map, since this map can be envisioned as arising from a string of light pulses impinging on a partially transmitting mirror of a ring cavity with a nonlinear dispersive medium [5, 8].

This work was partly supported by FAPESP under Grant 2012/24835-1 and by CNPq under Grants 303926/2010-4 and 302423/2011-7.

One of the worst drawbacks in chaos-based communication systems is the lack of robustness of chaotic synchronization with respect to noise and intersymbol interference (ISI) introduced by the channel. Even minor noise levels or simple linear distortions are enough to hinder communication [9, 10]. Many chaos-based communication systems tend to present prohibitive bit error rates in nonideal channels when compared to their conventional counterparts. Therefore, many approaches are based on the assumption of a rather ideal channel with a high signal-to-noise ratio (see, e.g., [4, 7, 11] and their references).

To mitigate the ISI introduced by the channel, it usual to consider an equalizer in the receiver. Equalization schemes applied to chaotic signals have been proposed in the literature for different approaches of message encoding (see, e.g., [12–16] and their references). However, we are not aware of works on equalization applied in the discrete-time domain for the chaotic modulation which feeds back the message into the CSG as in [6].

In this paper, we propose an adaptive equalization scheme for a discrete-time chaos-based communication system in which the message is fed back into the CSG. As CSG, we assume variants of the Ikeda maps. The paper is organized as follows. In Section 2, we describe a discrete-time version of the Wu and Chua's chaotic modulation [11], which besides a noisy and dispersive channel includes an adaptive equalizer. The synchronization of Ikeda maps is discussed in Section 3. A normalized least-mean-squares (NLMS) type algorithm applied to chaotic modulation is derived in Section 4. In Section 5, we obtain the interval of the step-size to ensure a local and weak stability of the proposed algorithm. Finally, Section 6 presents simulation results and in Section 7, we draft the conclusions.

2. PROBLEM FORMULATION

Fig. 1 shows the chaotic communication system under consideration, which is a discrete-time version of the one proposed by Wu and Chua [11]. In this scheme, the binary information signal $m(n) \in \{-1,\ +1\}$ is encoded by using the second component of the state vector $\mathbf{x}(n)$, i.e., $s(n) = m(n)x_2(n)$. Then, the signal $s(n)$ is fed back and transmitted through a communication channel, whose model is constituted by a transfer function $H(z)$ and additive white Gaussian noise

(AWGN). We assume an M-tap FIR adaptive equalizer, with input regressor vector $\mathbf{r}(n) = [\, r(n)\ r(n-1)\ \cdots\ r(n-M+1)\,]^T$ and output $\widehat{s}(n) = \mathbf{r}^T(n)\mathbf{w}(n-1)$, where $(\cdot)^T$ indicates transposition and $\mathbf{w}(n-1)$ is the equalizer weight vector. The equalizer must mitigate the intersymbol interference introduced by the channel and recover the encoded signal $s(n)$ with a delay of Δ samples. If transmitter and receiver synchronize, i.e., if $\widehat{\mathbf{x}}(n) \to \mathbf{x}(n)$, then using the output of the equalizer and the estimate of $x_2(n)$, the information signal can be decoded via

$$\widehat{m}(n) \triangleq \widehat{s}(n)/\widehat{x}_2(n), \tag{1}$$

where $\widehat{x}_2(n)$ is the second component of the state vector $\widehat{\mathbf{x}}(n)$. We also assume that there is a training sequence so that the error $e(n) = m(n-\Delta) - \widehat{m}(n)$ is used to adapt the equalizer coefficients in a supervised manner.

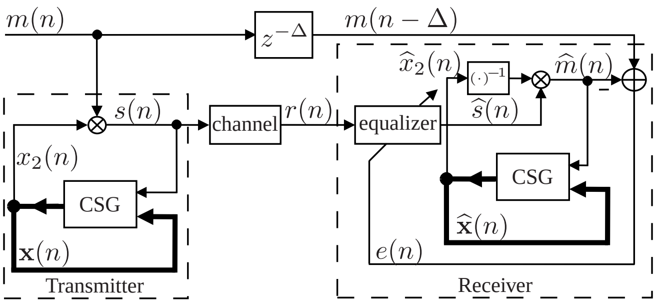

Fig. 1. Chaotic communication system with an adaptive equalizer.

In this paper, the Ikeda map is used in both CSGs in Fig. 1. Therefore, the equations governing the global dynamical system have the following form [5]

$$\mathbf{x}(n) = \mathbf{A}_t(n)\mathbf{x}(n-1) + [\, R\ \ 0\,]^T, \tag{2}$$
$$\widehat{\mathbf{x}}(n) = \mathbf{A}_r(n)\,\widehat{\mathbf{x}}(n-1) + [\, R\ \ 0\,]^T, \tag{3}$$

where $\mathbf{x}(n) \triangleq [x_1(n)\ x_2(n)]^T$, $\widehat{\mathbf{x}}(n) \triangleq [\widehat{x}_1(n)\ \widehat{x}_2(n)]^T$, and R is a constant, parameter of the Ikeda map. In the CSG of the transmitter, we have the matrix $\mathbf{A}_t(n)$ given by

$$\mathbf{A}_t(n) = C_2 \begin{bmatrix} \cos\theta_n & -\sin\theta_n \\ \sin\theta_n & \cos\theta_n \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & m(n-1) \end{bmatrix}, \tag{4}$$

where

$$\theta_n = C_1 - \frac{C_3}{1 + x_1^2(n-1) + x_2^2(n-1)m^2(n-1)} \tag{5}$$

and C_i, $i = 1,2,3$ are constant parameters of the Ikeda map. In the CSG of the receiver, we have

$$\mathbf{A}_r(n) = C_2 \begin{bmatrix} \cos\widehat{\theta}_n & -\sin\widehat{\theta}_n \\ \sin\widehat{\theta}_n & \cos\widehat{\theta}_n \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \widehat{m}(n-1) \end{bmatrix}, \tag{6}$$

where

$$\widehat{\theta}_n = C_1 - \frac{C_3}{1 + \widehat{x}_1^2(n-1) + \widehat{x}_2^2(n-1)\widehat{m}^2(n-1)}. \tag{7}$$

The system described by (2) is autonomous and is called *master*, whereas the one described by (3) depends on $\mathbf{x}(n-1)$ and is called *slave*. Notice that $\widehat{m}(n-1)$, defined in (1), depends on the master system, so the matrix $\mathbf{A}_r$ also depends on $\mathbf{x}(n-1)$.

3. COMPLETE SYNCHRONIZATION FOR AN IDEAL CHANNEL

The synchronization error is defined as $\boldsymbol{\xi}(n) \triangleq \widehat{\mathbf{x}}(n) - \mathbf{x}(n)$, which can be rewritten, using (2) and (3), as

$$\boldsymbol{\xi}(n) = [\mathbf{A}_r(n) - \mathbf{A}_t(n)]\, \boldsymbol{\xi}(n-1). \tag{8}$$

Master and slave are said *completely synchronized* if $\boldsymbol{\xi}(n) \to \mathbf{0}$ as n grows [17]. Consequently, they synchronize completely if the eigenvalues of $[\mathbf{A}_r(n) - \mathbf{A}_t(n)]$ satisfy $|\lambda_i(n)| < 1$, $i = 1,2$, for all n [18].

To prove theoretically that $|\lambda_{1,2}(n)| < 1$ for all n is not a simple task and some assumptions on the transmitted and recovered message are necessary, even when the channel is ideal. This occurs since in the Ikeda map $\mathbf{A}_t(n)$ and $\mathbf{A}_r(n)$ depend on $m(n-1)$ and $\widehat{m}(n-1)$, respectively. Therefore, we show next some numerical simulations to illustrate that the synchronization between master and slave can be achieved for an ideal channel, considering the usual parameters for the Ikeda map: $C_1 = 0.4$, $C_2 = 0.9$, $C_3 = 6$, and $R = 1$ [5].

Fig. 2 shows $|\lambda_{1,2}(n)|$ as a function of n in two situations: for $m(n) = 1$ and for $m(n) \in \{-1,\ +1\}$. As it can be noticed, we have synchronization for both cases, since $|\lambda_{1,2}(n)| < 1$ independently of the message considered. Furthermore, the largest Lyapunov exponent when $m(n) = 1$ is approximately 0.507, which means that $s(n)$ is chaotic [5]. For $m(n) \in \{-1,\ +1\}$, the generated signals still present the properties that characterize chaotic signals, although the Lyapunov exponent was not computed. We intend to pursue this matter elsewhere.

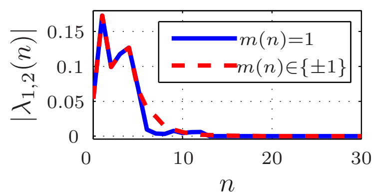

Fig. 2. Absolute value of the eigenvalues of $[\mathbf{A}_r(n) - \mathbf{A}_t(n)]$ along the iterations; Ikeda map ($\mathbf{x}(0) = \mathbf{0}$; $\widehat{\mathbf{x}}(0) = [0.1\ \ -0.1]^T$; $C_1 = 0.4$, $C_2 = 0.9$, $C_3 = 6$, and $R = 1$).

4. THE CHAOTIC NLMS ALGORITHM

To obtain a stochastic gradient algorithm to adapt the equalizer in the scheme of Fig. 1, we define the following instantaneous cost-function

$$\hat{J}(n) = e^2(n) = [m(n-\Delta) - \widehat{m}(n)]^2. \tag{9}$$

Computing the gradient of $\hat{J}(n)$ with respect to the coefficient vector $\mathbf{w}(n-1)$, we obtain

$$\nabla_{\mathbf{w}}\hat{J}(n) = 2e(n)\frac{\partial e(n)}{\partial \mathbf{w}(n-1)} = -2e(n)\frac{\partial \widehat{m}(n)}{\partial \mathbf{w}(n-1)}. \quad (10)$$

Assuming that $\widehat{x}_2(n) \neq 0$ for all n and taking into account the equalizer in the scheme of Fig. 1, (1) can be rewritten as

$$\widehat{m}(n) = \frac{\widehat{s}(n)}{\widehat{x}_2(n)} = \frac{\mathbf{r}^T(n)\mathbf{w}(n-1)}{\widehat{x}_2(n)}. \quad (11)$$

Using (11) and recalling that $\widehat{x}_2(n)$ depends only on $\widehat{\mathbf{x}}(n-1)$ and $\widehat{s}(n-1)$, which in turn do not depend on $\mathbf{w}(n-1)$, we arrive at

$$\nabla_{\mathbf{w}}\hat{J}(n) = -2\frac{e(n)}{\widehat{x}_2(n)}\frac{\partial \widehat{s}(n)}{\partial \mathbf{w}(n)} = -2\frac{e(n)}{\widehat{x}_2(n)}\mathbf{r}(n). \quad (12)$$

Thus, the update equation of the chaotic[1] LMS (cLMS) algorithm is given by

$$\mathbf{w}(n) = \mathbf{w}(n-1) + \mu\frac{e(n)}{\widehat{x}_2(n)}\mathbf{r}(n). \quad (13)$$

To obtain a normalized version of cLMS, we first define the *a posteriori* error as

$$e_p(n) = m(n-\Delta) - \frac{\mathbf{r}^T(n)\mathbf{w}(n)}{\widehat{x}_2(n)}. \quad (14)$$

Using (13), $e_p(n)$ can be rewritten as

$$\begin{aligned} e_p(n) &= m(n-\Delta) - \frac{\mathbf{r}^T(n)\left[\mathbf{w}(n-1) + \mu\dfrac{e(n)}{\widehat{x}_2(n)}\mathbf{r}(n)\right]}{\widehat{x}_2(n)} \\ &= e(n)\left[1 - \mu\frac{\|\mathbf{r}(n)\|^2}{\widehat{x}_2^2(n)}\right]. \end{aligned} \quad (15)$$

To enforce $e_p(n) = 0$ at each iteration n, we must select $\mu(n) = \widehat{x}_2^2(n)/\|\mathbf{r}(n)\|^2$. Introducing a fixed step-size $\tilde{\mu}$ to control the rate of convergence and a regularization factor δ to prevent division by zero in $\mu(n)$, and replacing the resulting step size in (13), we obtain the update equation of the cNLMS algorithm, i.e.,

$$\mathbf{w}(n) = \mathbf{w}(n-1) + \frac{\widetilde{\mu}}{\delta + \|\mathbf{r}(n)\|^2}\widehat{x}_2(n)e(n)\mathbf{r}(n). \quad (16)$$

Note that cNLMS depends not only on the estimation error $e(n)$, but also on $\widehat{x}_2(n)$. Since $\widehat{x}_2(n)$ depends nonlinearly on $\widehat{s}(n)$, cNLMS is a nonlinear version of NLMS. Moreover, the synchronization between master and slave in chaotic communication system depends on the mitigation of the intersymbol interference, which is the role played by the equalizer.

[1]We use the term *chaotic* for the algorithms derived here only for distinguishing them from the original versions of LMS and NLMS algorithms (see, e.g., [18]). The use of this term does not imply a chaotic behavior of the algorithms.

We prevent division by a value close to zero in the computation of $\widehat{m}(n)$, by making $\widehat{m}(n) = \text{sign}[\,\widehat{s}(n)\,\widehat{x}_2(n)\,]$ when $|\widehat{x}_2(n)| < \varepsilon$, where ε is a small positive constant and $\text{sign}[x] = -1$ if $x < 0$ or $\text{sign}[x] = 1$ if $x \geq 0$.

In order to ensure the stability of the algorithm and to avoid wrong estimates when $\widehat{x}_2(n)$ is too large, we introduce a bound for $\widehat{x}_2(n)$, i.e., if $|\widehat{x}_2(n)| > X$, we simply make $\widehat{x}_2(n) \leftarrow X\text{sign}[\widehat{x}_2(n)]$, where X is a positive constant. We do not observe performance degradation in different simulation scenarios, when we used $X = 100$. The proposed algorithm is summarized in Table 1.

Table 1. Summary of the cNLMS algorithm.

Initialize the algorithm by setting: $\mathbf{w}(-1) = \mathbf{0}$, $\widehat{\mathbf{x}}(0) = [\,0.1 \ \ -0.1\,]^T$, $\mathbf{b} = [\,R \ \ 0\,]^T$ δ, ε: small positive constants; X: large positive constant
For $n = 0, 1, 2, 3\ \ldots$, compute: $\widehat{s}(n) = \mathbf{r}^T(n)\mathbf{w}(n-1)$ if $\ |\widehat{x}_2(n)| > X$ $\widehat{x}_2(n) \leftarrow X\text{sign}[\,\widehat{x}_2(n)\,]$ end if $\ |\widehat{x}_2(n)| \leq \varepsilon$ $\widehat{m}(n) = \text{sign}[\,\widehat{s}(n)\,\widehat{x}_2(n)\,]$ else $\widehat{m}(n) = \dfrac{\widehat{s}(n)}{\widehat{x}_2(n)}$ end $e(n) = m(n-\Delta) - \widehat{m}(n)$ $\mathbf{w}(n) = \mathbf{w}(n-1) + \dfrac{\widetilde{\mu}_c}{\delta + \|\mathbf{r}(n)\|^2}\widehat{x}_2(n)e(n)\mathbf{r}(n)$ $\widehat{\theta}_{n+1} = C_1 - \dfrac{C_3}{1 + \widehat{x}_1^2(n) + \widehat{x}_2^2(n)\widehat{m}^2(n)}$ $\mathbf{A}_r(n{+}1) = C_2\begin{bmatrix}\cos\widehat{\theta}_{n+1} & -\sin\widehat{\theta}_{n+1}\\ \sin\widehat{\theta}_{n+1} & \cos\widehat{\theta}_{n+1}\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & \widehat{m}(n)\end{bmatrix}$ $\widehat{\mathbf{x}}(n+1) = \mathbf{A}_r\,(n+1)\,\widehat{\mathbf{x}}(n) + \mathbf{b}$ end

5. STABILITY CONDITIONS

Using (11), the update equation of cNLMS can be rewritten as

$$\begin{aligned} \mathbf{w}(n) = &\left[\mathbf{I} - \frac{\widetilde{\mu}}{\delta + \|\mathbf{r}(n)\|^2}\mathbf{r}(n)\mathbf{r}^T(n)\right]\mathbf{w}(n-1) \\ &+ \widetilde{\mu}\,\widehat{x}_2(n)m(n)\frac{\mathbf{r}(n)}{\delta + \|\mathbf{r}(n)\|^2}, \end{aligned} \quad (17)$$

where $\mathbf{I}$ is the identity matrix with dimensions $M \times M$. The matrix between brackets has $M-1$ eigenvalues equal to one and one eigenvalue equal to $\lambda_1 = 1 - \widetilde{\mu}\,\mathbf{r}^T(n)\mathbf{r}(n)/[\delta + \|\mathbf{r}(n)\|^2]$.

Noticing that

$$0 \leq \frac{\mathbf{r}^T(n)\mathbf{r}(n)}{\delta + \|\mathbf{r}(n)\|^2} < 1,$$

and for $\|\mathbf{r}(n)\|^2 \gg \delta$, $\mathbf{r}^T(n)\mathbf{r}(n)/(\delta + \|\mathbf{r}(n)\|^2) \approx 1$, in order to ensure $|\lambda_1| < 1$, we must choose $\widetilde{\mu}$ in the following interval

$$0 < \widetilde{\mu} < 2. \tag{18}$$

The norm of the second term of the r.h.s. of (17) is bounded, i.e.,

$$0 \leq \widetilde{\mu}\, |\widehat{x}_2(n)|\, |m(n)|\, \frac{\|\mathbf{r}(n)\|}{\delta + \|\mathbf{r}(n)\|^2} \leq \widetilde{\mu}\, X\, \frac{\sqrt{\delta}}{2\delta} < \infty.$$

Therefore, using (deterministic) exponential stability results for the LMS algorithm [19], we conclude that cNLMS is stable in a robust sense if $\widetilde{\mu}$ is chosen in the interval (18).

6. SIMULATION RESULTS

In all simulations, we assume the Ikeda map with $C_1 = 0.4$, $C_2 = 0.9$, $C_3 = 6$, and $R = 1$ [5, p.202]. The state vectors of (2) and (3) were initialized as $\mathbf{x}(0) = \mathbf{0}$ and $\widehat{\mathbf{x}}(0) = [0.1 \quad -0.1]^T$, respectively. Other initializations also allow equally good results in terms of synchronization when the equalizer mitigates reasonably well the intersymbol interference. Furthermore, we assume the transmission of a binary sequence $m(n) \in \{-1, 1\}$ and equalizers initialized as $\mathbf{w}(0) = \mathbf{0}$. For comparison, we also consider the system of Fig. 1 without equalizer, in which $\widehat{s}(n) = r(n)$.

We first assume that the encoded sequence $s(n)$ is transmitted through the infinite impulse response (IIR) channel $H_1(z) = 1/[1 + 0.6z^{-1}]$ with SNR $= 30$ dB and $\Delta = 0$. The sequence estimated via the cNLMS equalizer and the error after the decision device (both for one realization) are shown in Figs. 3-(b) and (c). The average of the two coefficients and the $\mathrm{MSE}(n) \triangleq \mathrm{E}\{e^2(n)\}$ along the iterations, estimated by an ensemble-average of 1000 runs, are shown respectively in Figs. 3-(d) and (e). In Fig. 3-(e), we also show the MSE curve for the case without equalizer. We can observe that the cNLMS converges in the mean to $\mathbf{w}_\mathrm{o} \approx [1 \quad 0.6]^T$, whose coefficients are shown as dashed lines in Fig. 3-(d). Therefore, the equalizer is working as expected since this solution mitigates the intersymbol interference, recovering properly the transmitted sequence, which can be confirmed through the errors after the decision device shown in Fig. 3-(c). The communication is completely lost in the case with no equalizer as shown if Figs. 3-(a) and (e).

Now, we assume that the encoded sequence $s(n)$ is transmitted initially through the real part of the telephonic channel of [20] and changed to its imaginary part at $n = 1000$, with SNR $= 30$ dB and $\Delta = 8$. The results for this case are shown in Fig. 4. cNLMS converges to the Wiener solution, whose coefficients are shown as dashed lines in Fig. 4-(d). It is important to notice that cNLMS is able to track the abrupt variation in the channel, leading approximately 600 iterations

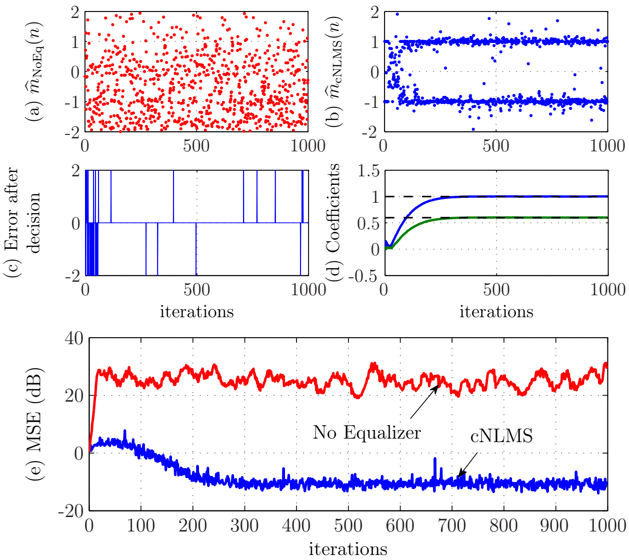

Fig. 3. Estimated sequence with (a) No equalizer and (b) cNLMS ($\mu = 0.1$, $\delta = 10^{-2}$, $\varepsilon = 0.1$); (c) Errors after decision; (d) Average of the coefficients of cNLMS and Wiener (dashed lines); (e) Estimated cMSE; average of 1000 runs, SNR $= 30$ dB; $M = 2$; $\Delta = 0$.

to achieve the steady-state again. The equalizer plays an important role to mitigate the intersymbol interference since the performance of the system without equalizer is much worse.

Considering SNR $= 30$ dB, $M = 5$, $\Delta = 3$, and the channel $H_3(z) = h_0 + z^{-1} + h_0 z^{-2}$, $0 \leq h_0 \leq 0.5$, we obtain BER curves as a function of h_0, as shown in Fig. 5. It is important to remark that in the case with no equalizer, the delay is due only to the channel. Therefore, we compared the recovered sequence with the transmitted one, assuming $\Delta = 1$ in this case. The smaller the value of h_0 the lower the intersymbol interference introduced by the channel. We can observe that cNLMS outperforms the case with no equalizer for $h_0 > 0$, providing a reasonable BER. The case with no equalizer achieves a BER equal to that of cNLMS only for the ideal channel ($h_0 = 0$). Note that the relatively high BER $\approx 10^{-2}$ in the left of Fig. 5 is only due to channel noise and reflects the extreme sensitivity of chaotic synchronization to noise. Obviously, the adaptive filters used here cannot tackle this problem. In the optical communication field, it is possible to obtain much higher values of SNR and consequently lower values of BER, as [6] did. The issue of channel equalization was successfully solved as shows the almost coincidence of the cNLMS and Wiener solution curves.

7. CONCLUSION

In this paper, we proposed a supervised equalization scheme based on the NLMS algorithm for recovering a binary sequence in chaos-based digital communication systems. Simulations show that the proposed algorithm can successfully permit chaotic communications. As far as we are concerned, this is the first adaptive scheme proposed for the chaotic modulation in which the message is fed back into the CSG. Although we considered the Ikeda map in the simulations, the cNLMS algorithm can be also used with other chaotic maps (e.g., Hénon map).

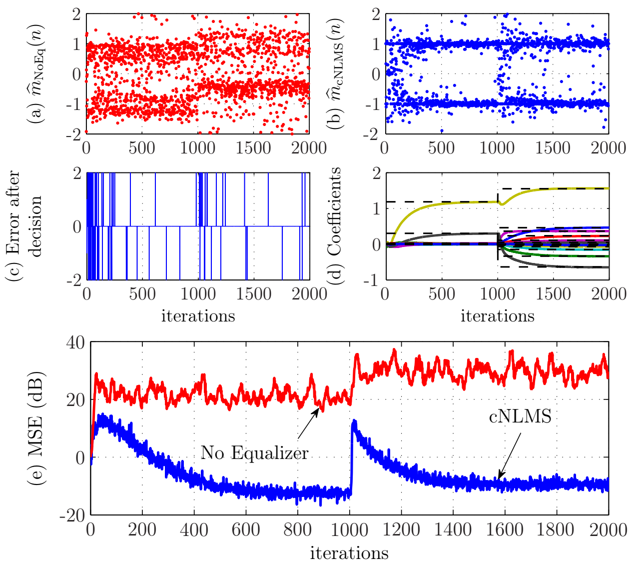

Fig. 4. Estimated sequence with (a) No equalizer and (b) cNLMS ($\mu = 0.5$, $\delta = 10^{-5}$, $\varepsilon = 0.1$); (c) Errors after decision; (d) Average of the coefficients of cNLMS and Wiener (dashed lines); (e) Estimated cMSE; average of 1000 runs, SNR $= 30$ dB; $M = 15$; $\Delta = 8$.

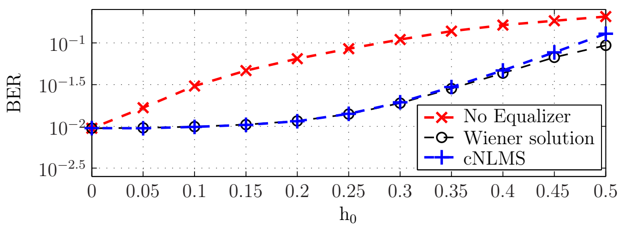

Fig. 5. Bit error rate as a function of the channel $H_3(z) = h_0 + z^{-1} + h_0 z^{-2}$ with SNR $=$ 30 dB; cNLMS ($\mu = 0.1$, $\delta = \varepsilon = 10^{-5}$).

Acknowledgement

The authors thank Dra. Maria D. Miranda for her useful suggestions.

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