A switching scheme between conventional and chaos-based communication systems

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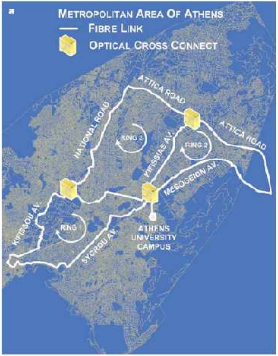
1. Introduction

- Chaotic signals present several interesting features for communications → candidates for spread spectrum communication systems
- Pecora and Carroll, 1990 → Chaotic Synchronization
- Many interesting ideias in the literature

- Optical communications: intrinsic nonlinear properties of lasers
  - Argyris et al., 2005: chaos based communication system (CBCS) using commercial fibre-optic link



1. Wu and Chua's communication system

- Simple way of using chaos for communications
- Verification of the convergence of the synchronization error is direct

Considering

$$\begin{aligned} \text{Master:} \quad \mathbf{x}(n+1) &= \mathbf{A}\mathbf{x}(n) + \mathbf{b} + \mathbf{f}(x_i(n)) \\ \text{Slave:} \quad \widehat{\mathbf{x}}(n+1) &= \mathbf{A}\widehat{\mathbf{x}}(n) + \mathbf{b} + \mathbf{f}(x_i(n)) \end{aligned}$$

Where:

- $\mathbf{x}(n)$ and $\widehat{\mathbf{x}}(n)$ are column vectors of size $K \times 1$
- $\mathbf{A}$ is a square matrix and $\mathbf{b}$ is a column vector, both constant
- $\mathbf{f}(\cdot)$: $\mathbb{R} \to \mathbb{R}^K$ is generally nonlinear, depending on only one component of $\mathbf{x}(n)$, with the form $\mathbf{f}(x_i(n)) = [\underbrace{0\ 0\ \cdots\ 0}_{i-1 \text{ zeros}}\ f\left(x_i(n)\right)\ \underbrace{0\ 0\ \cdots\ 0}_{K-i \text{ zeros}}]^T$

1. Wu and Chua's communication system (2)

$$\text{Master:} \quad \mathbf{x}(n+1) = \mathbf{A}\mathbf{x}(n) + \mathbf{b} + \mathbf{f}(x_i(n))$$
$$\text{Slave:} \quad \widehat{\mathbf{x}}(n+1) = \mathbf{A}\widehat{\mathbf{x}}(n) + \mathbf{b} + \mathbf{f}(x_i(n))$$

Synchronization error:

$$\mathbf{e}(n) \triangleq \widehat{\mathbf{x}}(n) - \mathbf{x}(n)$$
$$\mathbf{e}(n+1) = \mathbf{A}\mathbf{e}(n)$$

If the eigenvalues λ_i of $\mathbf{A}$ satisfy $|\lambda_i| < 1, 1 \leq i \leq K$:

$$\mathbf{e}(n) \to \mathbf{0} \Rightarrow \text{Chaotic synchronization}$$

1. Wu and Chua's communication system (3)

$$\text{Master:} \quad \mathbf{x}(n+1) = \mathbf{A}\mathbf{x}(n) + \mathbf{b} + \mathbf{f}(x_i(n))$$
$$\text{Slave:} \quad \widehat{\mathbf{x}}(n+1) = \mathbf{A}\widehat{\mathbf{x}}(n) + \mathbf{b} + \mathbf{f}(x_i(n))$$

Transmitting information using Wu and Chua's system

- Encoding $m(n)$ using the i-th component of $\mathbf{x}(n)$:

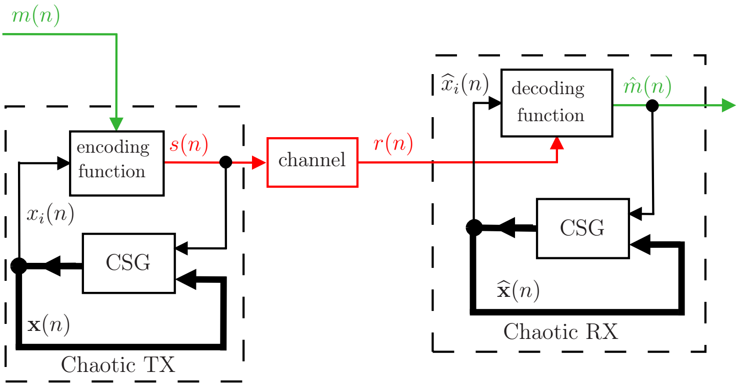
$$s(n) = c(x_i(n), m(n))$$

- We can recover

$$m(n) = c^{-1}\left(\widehat{x}_i(n), s(n)\right)$$

$$\text{Transmitter:} \quad \mathbf{x}(n+1) = \mathbf{A}\mathbf{x}(n) + \mathbf{b} + \mathbf{f}(s(n))$$
$$\text{Receiver:} \quad \widehat{\mathbf{x}}(n+1) = \mathbf{A}\widehat{\mathbf{x}}(n) + \mathbf{b} + \mathbf{f}(s(n))$$

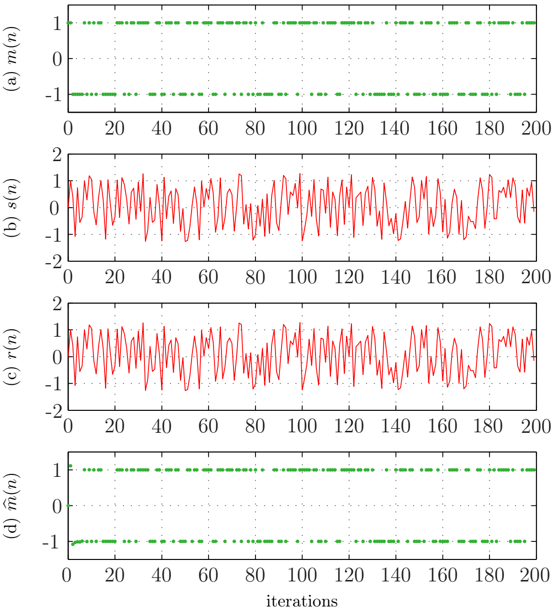
1. Wu and Chua's communication system (4)



$$\mathbf{x}(n+1) = \mathbf{A}\mathbf{x}(n) + \mathbf{b} + \mathbf{f}(s(n))$$
$$\widehat{\mathbf{x}}(n+1) = \mathbf{A}\widehat{\mathbf{x}}(n) + \mathbf{b} + \mathbf{f}(r(n))$$

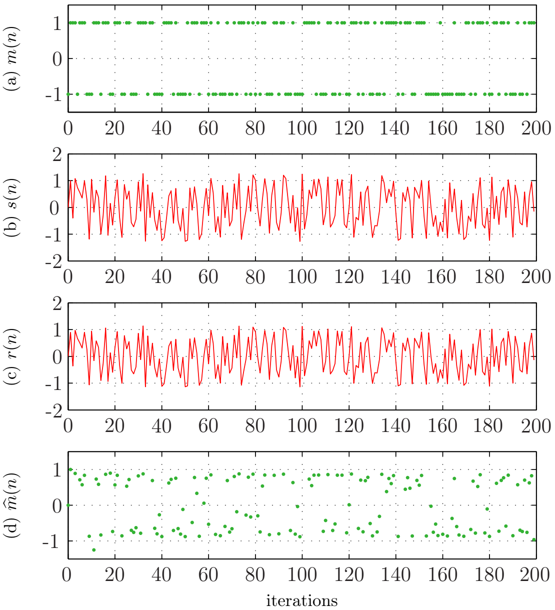
1. Wu and Chua's communication system (5)

- Hénon map as CSG
- Ideal channel
- $s(n) = m(n)x_1(n)$ e $\widehat{m}(n) = r(n)/\widehat{x}_1(n)$



1. Wu and Chua's communication system (6)

- Channel $H(z) = 0.9$
  $r(n) = 0.9s(n)$
- Chaotic synchronization sensible to non-ideal channels



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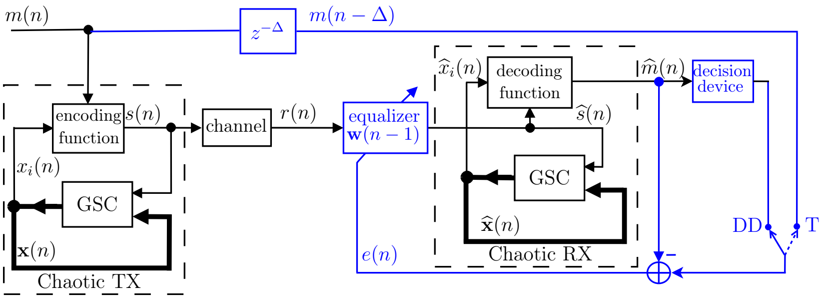
2. CBCS with equalizer



- $\widehat{s}(n) = \mathbf{r}^T(n)\mathbf{w}(n-1) \qquad \widehat{m}(n) = c^{-1}(\widehat{x}_i(n), \widehat{s}(n)) \qquad e(n) = m(n-\Delta) - \widehat{m}(n)$
- We assume there is a training sequence $m(n) \Rightarrow$ Training mode
- After training, when transmitting an actual message, $m(n-\Delta)$ is given by the output of a decision device applied to $\widehat{m}(n) \Rightarrow$ Decision-directed mode

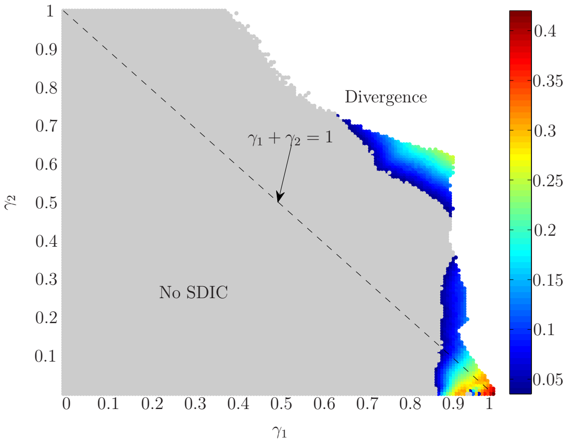
2. Encoding function

$$s(n) = \gamma_1 \, x_1(n) - \gamma_2 \, [m(n) + 1] \, \text{sign}[\gamma_1 \, x_1(n)]$$

$$\widehat{m}(n) = \frac{\gamma_1 \widehat{x}_1(n) - \widehat{s}(n)}{\gamma_2 \text{sign}[\gamma_1 \widehat{x}_1(n)]} - 1$$

Properties:

- If $\gamma_1 = 0.9$ and $\gamma_2 = 0.3$, s(n) is chaotic (presents SDIC)
- If $\gamma_1 = 0$ and $\gamma_2 = 1$, the system is equivalent to a conventional communication system with no chaos
- It is possible to change γ_1 and γ_2 online, considering the change is synchronized in transmitter and receiver



2. Equalization Algorithm

Objective

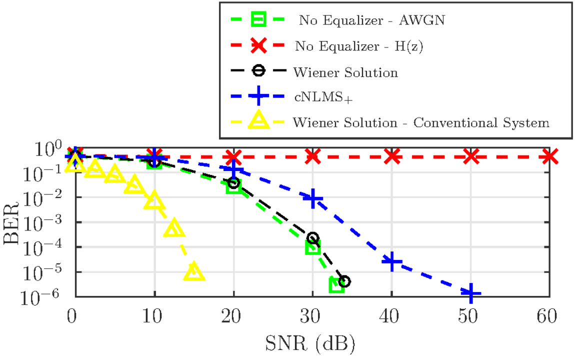
- Minimize: $\widehat{C}(n) = e^2(n)$
- $e(n) = m(n - \Delta) - \widehat{m}(n)$
- $\widehat{m}(n) = \dfrac{\gamma_1(n)\widehat{x}_1(n) - \widehat{s}(n)}{\gamma_2(n)\mathrm{sign}[\gamma_1(n)\widehat{x}_1(n)]} - 1$
- Assumption: $\widehat{x}_1(n)$ is independent of $\mathbf{w}(n-1)$

cNLMS$_+$ algorithm [Candido, Silva, Eisencraft - DINCON 2015]

$$\mathbf{w}(n) = \mathbf{w}(n-1) - \frac{\widetilde{\mu}}{\delta + \|\mathbf{r}(n)\|^2}\gamma_2(n)\mathrm{sign}[\gamma_1(n)\widehat{x}_1(n)]e(n)\mathbf{r}(n)$$

For convergence $\rightarrow 0 < \widetilde{\mu} < 2$

2. Simulation - Equalization



AWGN channel and $H(z) = 0.25 + z^{-1} + 0.25z^{-2}$; $M = 21$; delay $\Delta = 11$; cNLMS$_+$ with $\widetilde{\mu} = 0.02$, $\delta = 10^{-5}$; 6×10^5 bits for convergence and 7×10^5 bits for BER calculation; Training mode

Despite their properties, CBCSs have a worse performace in terms of BER, when compared to conventional communication systems.

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3. Switching Scheme

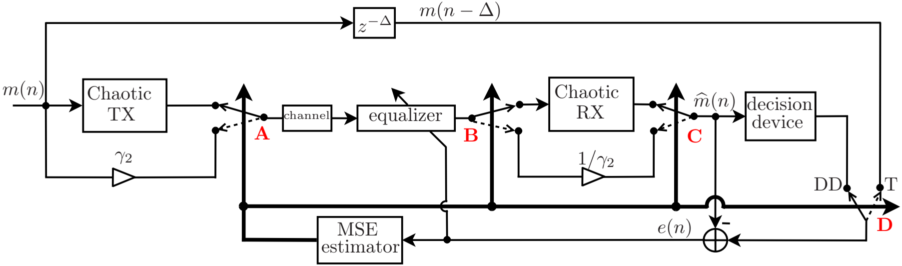
Inspired by Wi-Fi technology, which switches modulation depending on the communication channel quality, we use

$$\mathrm{MSE}(n_0) = \frac{1}{L} \sum_{k=n_0}^{n_0+L-1} e^2(k),$$

with $n_0 = 0, L, 2L, \cdots$ as a trigger, to switch between:

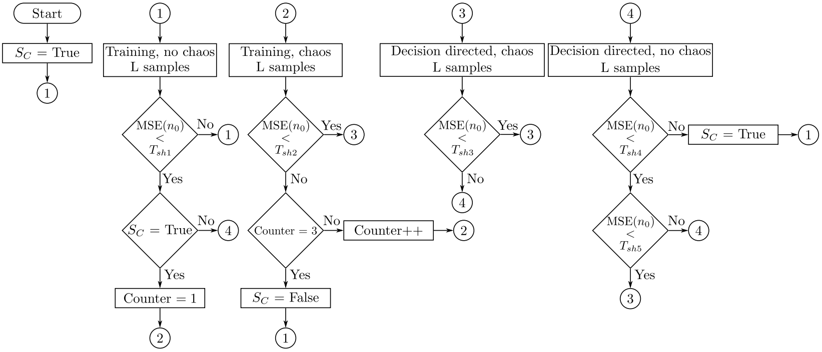
- Chaos based communication system $\Rightarrow \gamma_1 = 0.9$ and $\gamma_2 = 0.3$
- Conventional communication system $\Rightarrow \gamma_1 = 0$ and $\gamma_2 = 0.4562$ (maintaining $s(n)$ power)
- and their respective trainning and decision directed modes

3. Switching Scheme (2)



- **A**, **B**, and **C** switch between Chaos or Conventional communication system
- **D** switches between Training (T) or Decision-derected (DD) mode

3. Switching Scheme (3)



Operation modes:

- 1: Training, no chaos
- 2: Training, chaos
- 3: DD, chaos
- 4: DD, no chaos

- Switching based on 5 thresholds
- Chaos, DD can be achieved:
  - $1 \to 2 \to 3$
  - $1 \to 4 \to 3$
- In case of failure, Conventional DD is maintained

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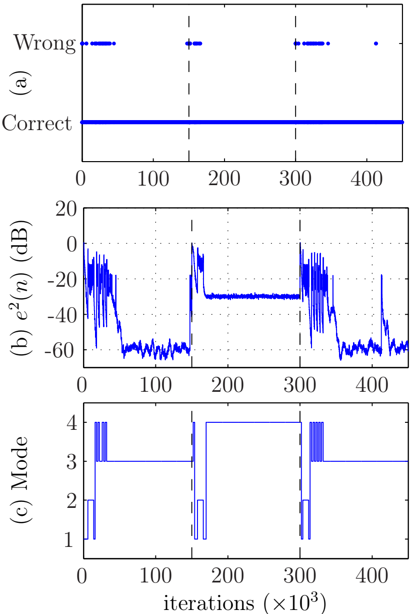
4. Simulations



Channels [Picchi and Prati, 1987]:

- from $n = 0$ to $n = 1.5 \times 10^5$ $\Rightarrow$ low ISI channel ("easy to equalize")
- from $n = 1.5 \times 10^5$ to $n = 3 \times 10^5$ $\Rightarrow$ higher ISI channel ("harder to equalize")
- from $n = 3 \times 10^5$ to $n = 4.5 \times 10^5$ $\Rightarrow$ back to low ISI channel

$L = 2000$

Modes:

- 1: Training, no chaos
- 2: Training, chaos
- 3: DD, chaos
- 4: DD, no chaos

We assume switching is synchronized in transmitter and receiver

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5. Preliminary Conclusions

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- The switching sequence that seems more appropriate is: Training, No chaos $\rightarrow$ DD, no chaos $\rightarrow$ DD, chaos;
- The thresholds that drive the switching scheme may be dependent on the channel;
- A BER threshold may be more interesting to use as a trigger to the switching scheme;
- We may study ways to synchronize the switching between transmitter and receiver (maybe using acknowledgments).

Thank you!

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